BEE402-ELECTRICAL NETWORK ANALYSIS AND SYNTHESIS

S.Sherine

Asst.Prof/EEE,BIHER

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Property 1. L-C immittance function

- 1. $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of odd to even or even to odd polynomials.

  - Consider the impedance Z(s) of passive one-port network.

$$Z(s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$$

(M is even N is odd)

As we know, when the input current is *I*, the average power dissipated by one-port network is zero:

Average Power= $\frac{1}{2}\mathrm{Re}[Z(j\omega)]|I|^2$ =0

$$EvZ\ (s) = \frac{M_1(s)M_2(s) - N_1(s)N_2(s)}{M_2(s)^2 + N_2(s)^2} = 0$$

$$M_1(j\omega)M_2(j\omega) - N_1(j\omega)N_2(j\omega) = 0$$

$$M_1 = 0 = N_2 \quad \text{OR} \quad M_2 = 0 = N_1$$

$$Z(s) = \frac{M_1}{N_2}, Z(s) = \frac{N_1}{M_2}$$

Z(s) or Y(s) is the ratio of even to odd or odd to even!!

Property 2. L-C immittance function

- 2.The poles and zeros are simple and lie on the $j\omega$ axis.

$$Z(s) = \frac{M_1}{N_2}, Z(s) = \frac{N_1}{M_2}$$

  - Since both M and N are Hurwitz, they have only imaginary roots, and it follows that the poles and zeros of Z(s) or Y(s) are on the imaginary axis.
  - Consider the example $$Z(s) = \frac{a_4 s^4 + a_2 s^2 + a_0}{b_5 s^5 + b_3 s^3 + b_1 s}$$

$$Z(s)=\frac{a_4 s^4+a_2 s^2+a_0}{b_5 s^5+b_3 s^3+b_1 s}$$

In order for the impedance to be positive real → the coefficients must be real and positive. $j\omega$

Impedance function **cannot have multiple poles or zeros** on the

axis.

The highest powers of the numerator and the denominator polynomials can differ by, at most, unity.

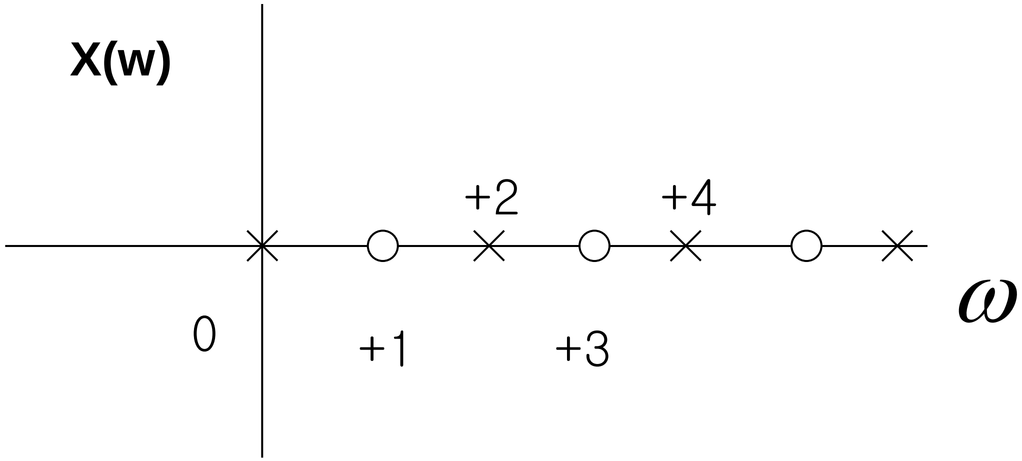
∞

Ex) highest order of the numerator : 2n -> highest order of the denominator can either be 2n-1 (simple pole at s=) or

the order can be 2n+1 (simple zero at s= ∞).

Property 3. L-C immittance function

- 3.The poles and zeros interlace on the $j\omega$ axis.



Highest power: 2n -> next highest power must be 2n-2

They cannot be missing term. Unless?

if $b_5 s^5 + b_1 s = 0$ → **s=0,** $s_k = (\frac{b_1}{b_5})^{1/4} e^{i(2k-1)\pi/4}$

We can write a general L-C impedance or admittance as

$$Z(s)=\frac{K(s^2+\omega_1{}^2)(s^2+\omega_3{}^2)\dots(s^2+\omega_i{}^2)\dots}{s(s^2+\omega_2{}^2)(s^2+\omega_4{}^2)\dots(s^2+\omega_j{}^2)\dots}$$

$$Z(s)=\frac{K_0}{s}+\frac{2K_2 s}{s^2+\omega_2{}^2}+\frac{2K_4 s}{s^2+\omega_4{}^2}+\dots+K_\infty s$$

Since these poles are all on the jw axis, the residues must be real and positive in order for Z(s) to be positive real .

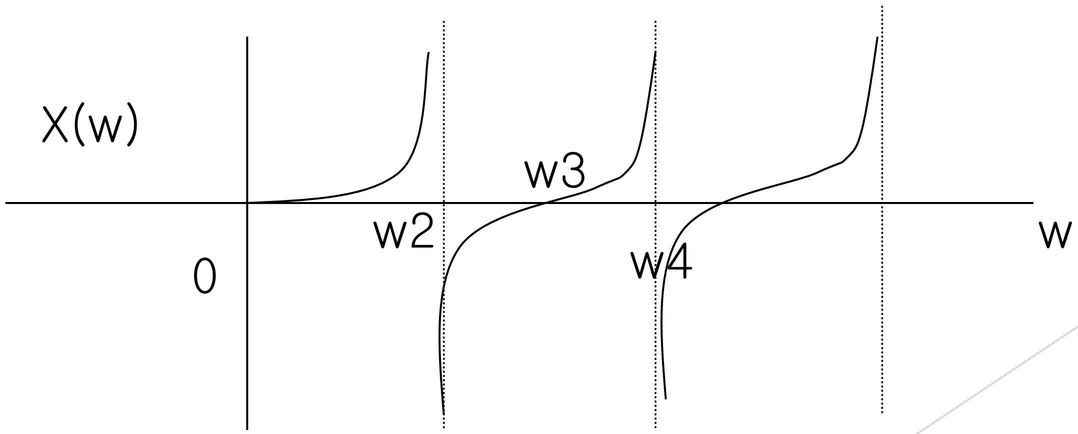
S=jw → Z(jw)=jX(w) (no real part)

$$\frac{dX(\omega)}{d\omega} = \frac{K_0}{\omega^2} + K_\infty + \frac{K_2(\omega^2 + \omega_2{}^2)}{(\omega_2{}^2 + \omega^2)} + \dots$$

Since all the residues Ki are positive, it is eay to see that for an L-C function

$$\frac{dX(\omega)}{d\omega} \geq 0$$

Ex) $Z(s) = \dfrac{Ks(s^2 + \omega_3{}^2)}{(s^2 + \omega_2{}^2)(s^2 + \omega_4{}^2)}$ $jX(\omega) = +j\dfrac{K\omega(-\omega^2 + \omega_3{}^2)}{(-\omega^2 + \omega_2{}^2)(-\omega^2 + \omega_4{}^2)}$



Properties 4 and 5. L-C immittance function

- The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.
- There must be either a zero or a pole at the origin and infinity.

Summary of properties

1. $Z_{LC}(s)$ or Y_{LC} (s) is the ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple and lie on the jw axis

3. The poles and zeros interlace on the jw axis.

4. The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.

5. There must be either a zero or a pole at the origin and infinity.

Examples

$$Z(s) = \frac{Ks(s^2+4)}{(s^2+1)(s^2+3)}$$

$$Z(s) = \frac{s^5+4s^3+5s}{3s^4+6s^2}$$

$$Z(s) = \frac{K(s^2+1)(s^2+9)}{(s^2+2)(s^2+10)}$$

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

Synthesis of L-C Driving point immittances

- L-C immittance is a positive real function with poles and zeros on the jw axis only.

$$Z(s)=\frac{K_0}{s}+\frac{2K_2 s}{s^2+\omega_2^{\ 2}}+\frac{2K_4 s}{s^2+\omega_4^{\ 2}}+\ldots+K_\infty s$$

- The synthesis is accomplished directly from the partial fraction .
- F(s) is impedance -> then the term K_0 / s capacitor of 1/K farads

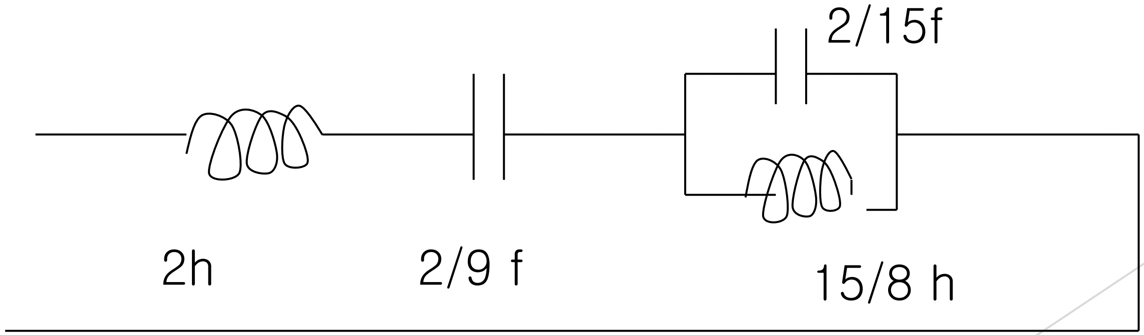
  the K(infinite)s is an inductance of K(infinite) henrys.

For Z(s) partial fraction

$2K_i s/(s^2+\omega_i^2)$ Is a parallel tank capacitance and inductance.

$1/2K_i$, $2K_i/\omega_i^2$

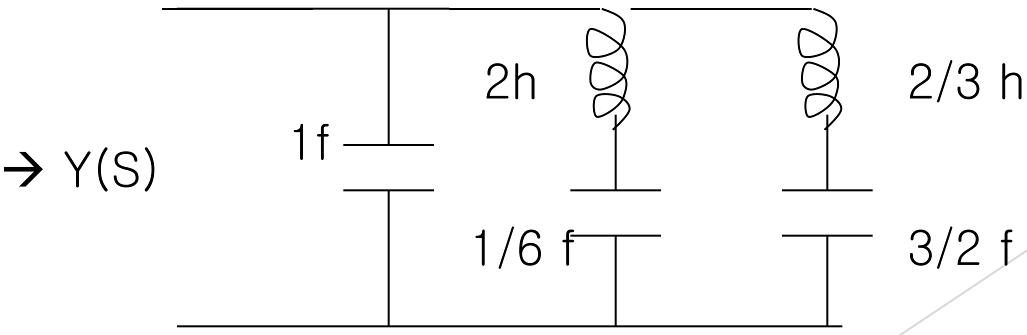
$$Z(s)=\frac{2(s^2+1)(s^2+9)}{s(s^2+4)}=2s+\frac{\frac{9}{2}}{s}+\frac{\frac{15}{2}s}{s^2+4}$$



For Y(s) partial fraction

- In admittance

$$Y(s)=\frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}=s+\frac{\frac{1}{2}s}{s^2+3}+\frac{\frac{3}{2}s}{s^2+1}$$



Another methodology

- Using property 4 “The highest powers if numerator and denominator must differ by unity; the lowest powers also differ by unity.”
  - Therefore, there is always a zero or a pole at s=infinite .
  - suppose Z(s) numerator:2n ,denominator:2n-1
  - this network has pole at infinite. -> we can remove this pole by removing an impedance $L_1 s$

$$Z_2(s) = Z(s) - L_1 s$$

- Degree of denominator : 2n-1 numerator:2n-2
  - $Z_2(s)$ has zero at s=infinite.
  - $Y_2(s) = 1/Z_2(s)$, $\rightarrow$ $Y_3(s) = Y_2(s) - C_2 s$

- This infinite term removing process continue until the remainder is zero.
- Each time we remove the pole, we remove an inductor or capacitor depending upon whether the function is an impedance or an admittance.
- → Final synthesized is a ladder whose series arms are inductors and shunt arms are capacitors.

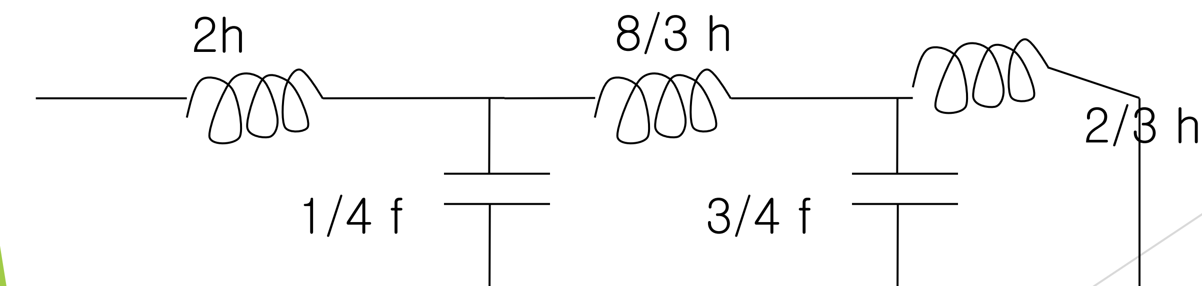
$$Z(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3}$$

$$Z_2(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3} - 2s = \frac{4s^3 + 10s}{s^4 + 4s^2 + 3}$$

$$Y_3(s) = \frac{s^4 + 4s^2 + 3}{4s^3 + 10s} - \frac{1}{4}s = \frac{\frac{3}{2}s^2 + 3}{4s^3 + 10s}$$

$$Z_4(s) = Z_3 - \frac{8}{3}s = \frac{2s}{\frac{3}{2}s^2 + 3}$$

2h



- This circuit (Ladder) called as Cauer

  because Cauer discovered the continues fraction method.

- Without going into the proof of the statement m in can be said that both the Foster and Cauer form gice the minimum number of elements for a specified L-C network.

Example of Cauer Method

$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$